Trisect a Line Segment (2)

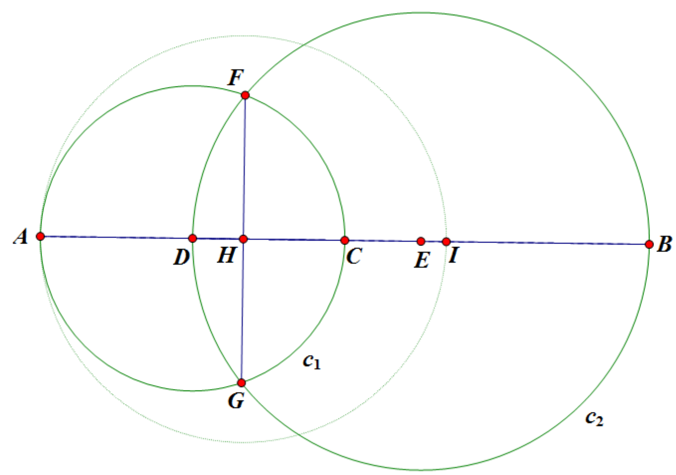
Method:

1) Let's take a line segment AB
2) Find the midpoint of AB, label it as C
3) Find midpoint of AC, label it D
4) Construct a circle D with radius AD, label it c_1
5) Find midpoint of BD, label it E
6) Construct a circle E with radius DE, label it c_2
7) Label the intersections of two circles as F and G
8) Construct a segment FG.
9) Segment FG intersects segment AB, label the intersection as H
10) Construct circle H with radius AH
11) Circle H intersects segment AB, label the intersection as I.

Claim: Points H and I trisects segment AB.

Proof:

Let's define the length of segment AB = b, where b$\in \mathbb{R}$. So, the coordinates of the point A is (0,0) and coordinate for point B is (b,0). Hence we suppose that the segment AB lie in on x-axis on a xy-plane.



So the equation of circle c_1 with center at D $(\frac{b}{4}, 0)$ and radius = $\frac{b}{4}$

$(x-\frac{b}{4})^2 + y^2 = (\frac{b}{4})^2$

Similarly, the equation of circle c_2 with center E $(\frac{5b}{4}, 0)$ and radius = $\frac{3b}{8}$

$(x-\frac{5b}{4})^2 + y^2 = (\frac{3b}{8})^2$

Since points F and G are the intersections of the two circles and segment FG intersects segment AB at point H, we can solve this two equations to find the coordinate of H. Subtracting the two equations yields,

$$(x-\frac{b}{4})^2 - (x-\frac{5b}{4})^2 = (\frac{b}{4})^2 - (\frac{3b}{8})^2$$

Solving the equations yields, $x = \frac{b}{3}$. Thus the coordinate for H is $(\frac{b}{3}, 0)$. Hence AH = $\frac{b}{3}$ or $\frac{1}{3}$ of the segment AB. Similarly, we can prove that HI = IB = $\frac{1}{3}$.